Lecture 3: Motion and Gravity

Claire Max

April 8th, 2014

Astro 18: Planets and Planetary Systems

UC Santa Cruz

Practicalities, 1

- No lecture this Thursday April 10th
- The next lecture will be Tuesday April 15th

Practicalities, 2

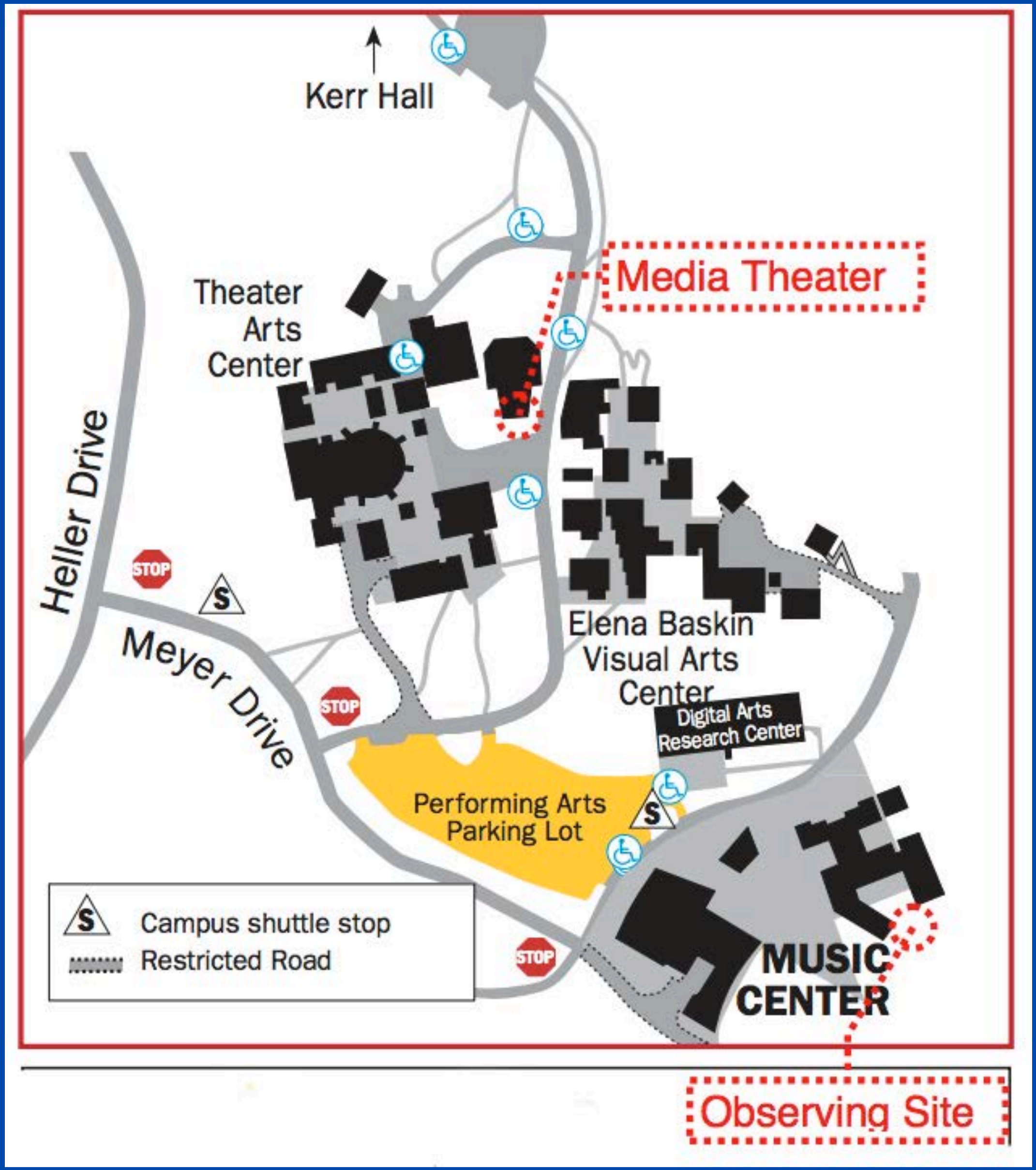
- I still haven't received Homework 1 from 5 of you. Please send ASAP. You can find the assignment on the class website, http://www.ucolick.org/~max/Astro18-2014/Astro18.html

- Signing up for this course:
  - If you are not yet officially enrolled but want to be, see me during the break.

- I will post Homework 2 and the reading assignment on the class website by this Thursday at the latest

First stargazing night: next Monday, April 14th

- A total eclipse of the moon
- Starts at 11:30 pm. Try to be there by 10 or 10:30 pm so we can also look at Jupiter and Saturn and Mars.
- Look for me when you arrive, so you can sign in.
- Recall that attending at least one stargazing evening is a mandatory part of this class. There will be others, but this is the only lunar eclipse!
- Location: Large meadow just below music center. Map: http://www.astro.ucsc.edu/news-events/astro_club1/UCSC%20Star%20Gazing%20Map.pdf



- Please check http://www.astro.ucsc.edu/astronomy_club before you come, in case of rain.
- I will also send out an email if the stargazing is cancelled
- Bring warm clothes and a flashlight

Outline of this lecture

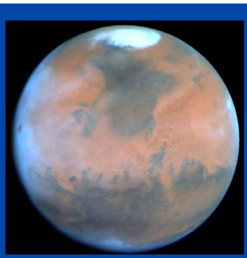
- Gravity: historical development of concepts
- Velocity and acceleration
- Gravity: Newton's laws, orbits

Please remind me
to take a break at
12:45 pm!

The Main Point

- Motions of planets, their moons, asteroids, and comets can be very accurately predicted because of underlying physical laws
  - Laws of planetary motion discovered by Kepler
  - Laws of gravity and conservation of momentum discovered by Newton
    - » Kepler's Laws can be derived from Newton's Laws, which are more general

History: How did astronomical observations benefit ancient societies?



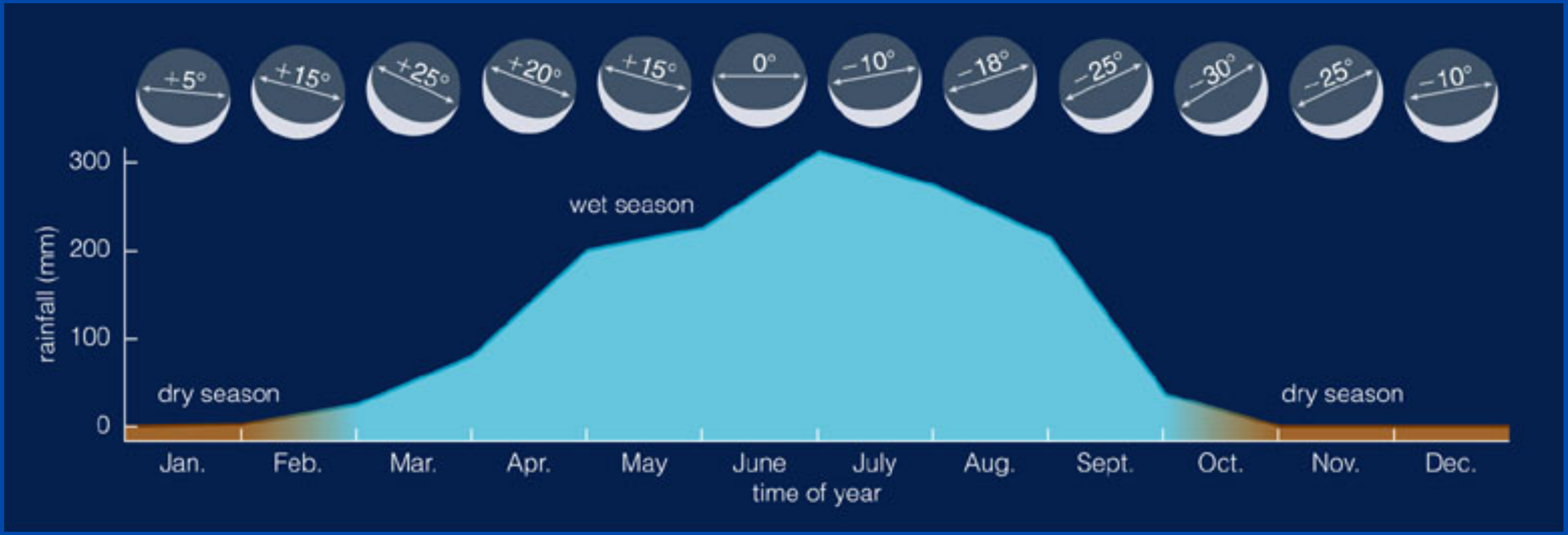
- Keeping track of time and seasons
  - for practical purposes (e.g. agriculture)
  - for religious and ceremonial purposes

- Aid to navigation
  - Ancient Polynesians a spectacular example

Ancient Polynesian Navigators

- Ancient Polynesians used celestial navigation (positions of the stars) to sail thousands of miles over open ocean
- To the Society Islands, the Marquesas, Easter Island in the east, the Hawaiian Islands in the north, and New Zealand in the southwest.

A stunning example of ancient African astronomy



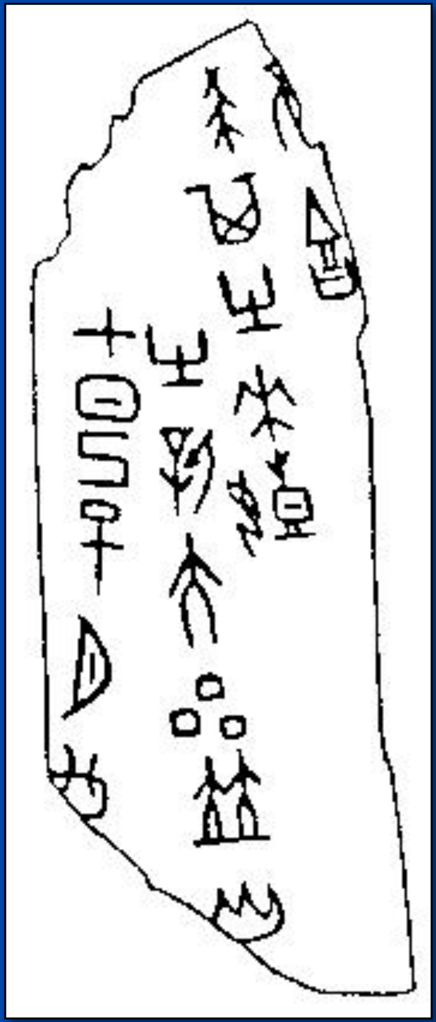
Ancient people of central Africa (6500 BC) could predict seasons from orientation of crescent moon!

Important for agriculture!

What did ancient civilizations achieve in astronomy?

- Daily timekeeping
- Tracking the seasons and calendar
  - Key for Agriculture, religious observence
- Monitoring lunar cycles
- Monitoring planets and stars
- Predicting eclipses
- Naming the stars and constellations
- And more…

China: Earliest known records of supernova explosions (1400 B.C.)

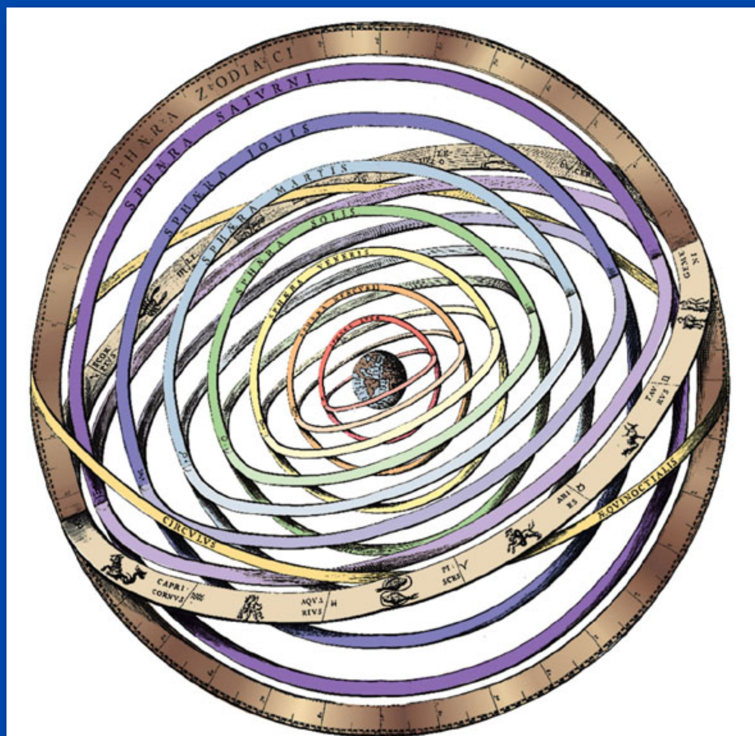


"On the Jisi day, the 7th day of the month, a big new star appeared in the company of the Ho star."

"On the Xinwei day the new star dwindled."

Bone or tortoise shell inscriptions from the 14th century BC (!)

Why does modern science trace its roots to the Greeks?

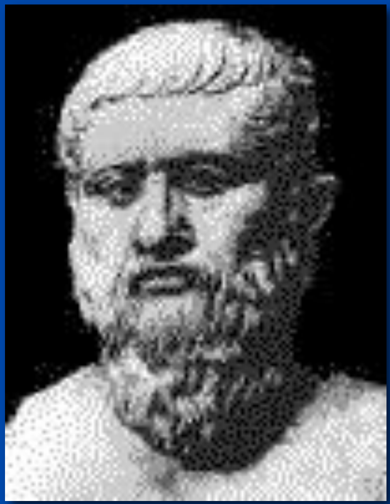


Greek geocentric model of the Solar System (c. 400 B.C.)

- **Greeks were the first people known to make models of nature.**
- **They tried to explain patterns in nature without resorting to myth or the super-natural.**

How did the Greeks explain planetary motion?

Underpinnings of the Greek geocentric* model:



Plato

Earth is at the center of the universe

Heavens must be "perfect": Objects moving on perfect spheres or in perfect circles.

Aristotle

* Sun, stars, planets rotate around the Earth

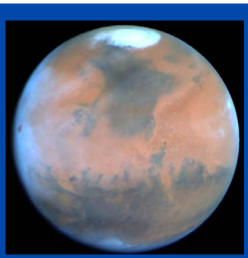
The most sophisticated geocentric model was that of Ptolemy

Ptolemy (A.D. 100-170)

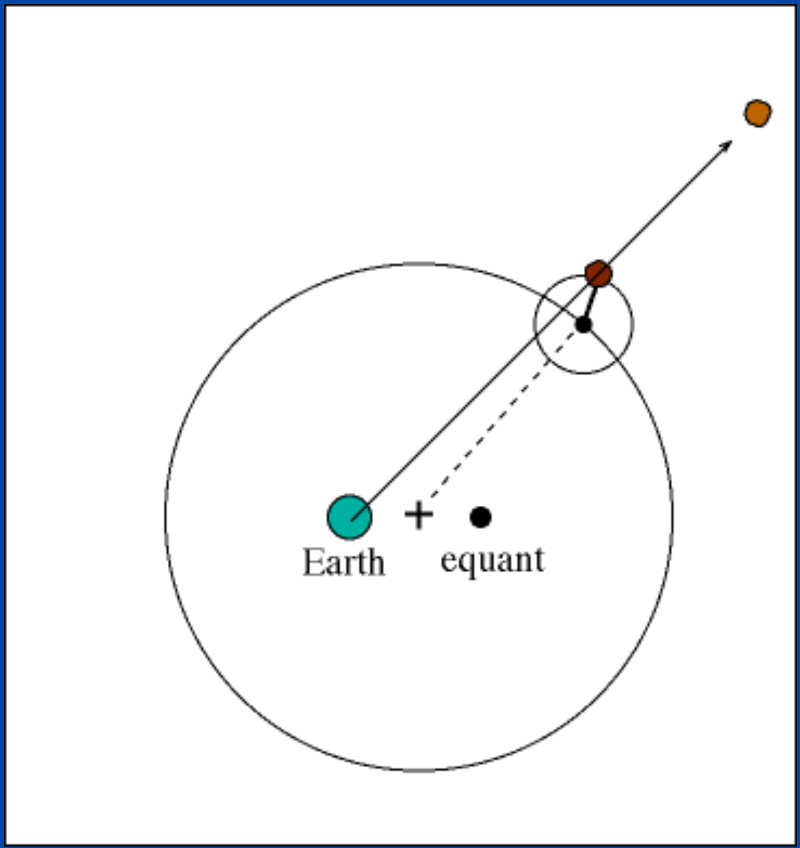
The Ptolemaic model:

- Hugely successful
- Sufficiently accurate to remain in use for 1,500 years!
- Arabic translation of Ptolemy's work named Almagest ("the greatest compilation")

Ptolemy: lived 100 - 170 AD in Alexandria Egypt (a Greek city at the time)

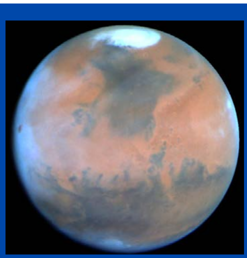


- Earth at center of universe
- Stars move on spheres
- Planets move on spheres within spheres
- Seems implausible today, but gave very good predictions of positions of planets, moon, etc given the quality of data available



- Epicycle animation

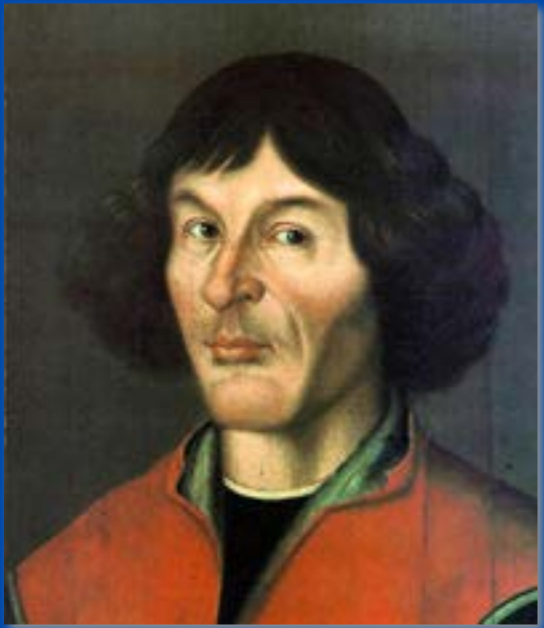
The Islamic world built on and enhanced Greek knowledge for 800 years



- Golden Age of Arabic-Islamic science
- 500 years: from 8th to 13th centuries C.E. -- longer than modern Western science
- Al-Mamun's House of Wisdom in Baghdad a great center of learning around A.D. 800
- Arabs invented algebra, made major strides in medicine, anatomy, pharmacology, astronomy
- With the fall of Constantinople (Istanbul) in 1453, Eastern scholars headed west to Europe, carrying knowledge that helped ignite the European Renaissance.

Copernicus

- Nicolaus Copernicus (1473 - 1543), Poland
- First European astronomer to formulate a modern heliocentric* theory of the solar system.
- A mathematician, astronomer, jurist, physician, classical scholar, Catholic cleric, governor, administrator, diplomat, economist and military leader (!)



***Heliocentric: sun at center of Solar System**

Copernicus, continued

- Amid Copernicus' extensive other responsibilities, astronomy was little more than a hobby.
- Heliocentric (sun at the center) theory had been formulated by Greeks and Muslims centuries before Copernicus.
- But his reiteration that the sun (rather than the Earth) is at the center of the solar system is considered among the most important landmarks in the history of western science.

1500's AD: Two models existed that made predictions about orbits of planets

- Ptolemy: Earth at center of "universe"
- Copernicus: Sun at center of "universe"

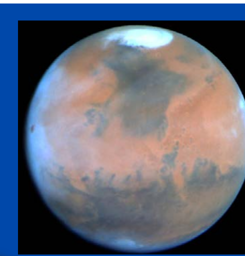
Copernicus' Sun-centered model was not much more accurate than Ptolemy's

- By that time, Ptolemaic model was noticeably inaccurate
- Copernicus concluded that a Sun-centered Solar System could predict planetary motions more easily
- But he believed that planetary orbits must be circles (they aren't)
- Hence his model's predictions were not much more accurate than Ptolemy!

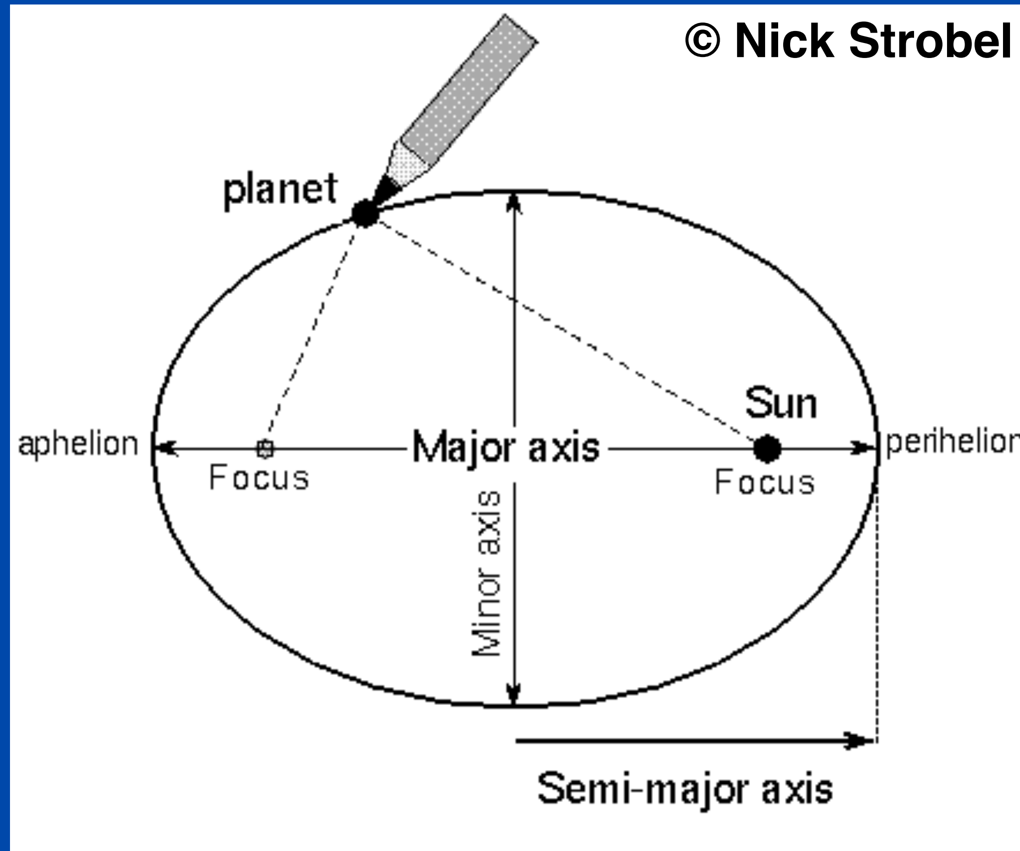
What was needed was high quality data

- **Tycho Brahe - Danish, 1546-1601**
  - Very accurate naked-eye observations of positions of planets and stars
  - Persisted for three decades, kept careful records
  - Couldn't explain why his data looked the way they did, but he hired a young apprentice who did explain it:
- **Johannes Kepler - German, 1571-1630**
  - Realized that if he tried to predict position of Mars using circular orbits, it didn't fit data
  - Abandoned circular orbits, developed a theory using elliptical orbits

Kepler's first law



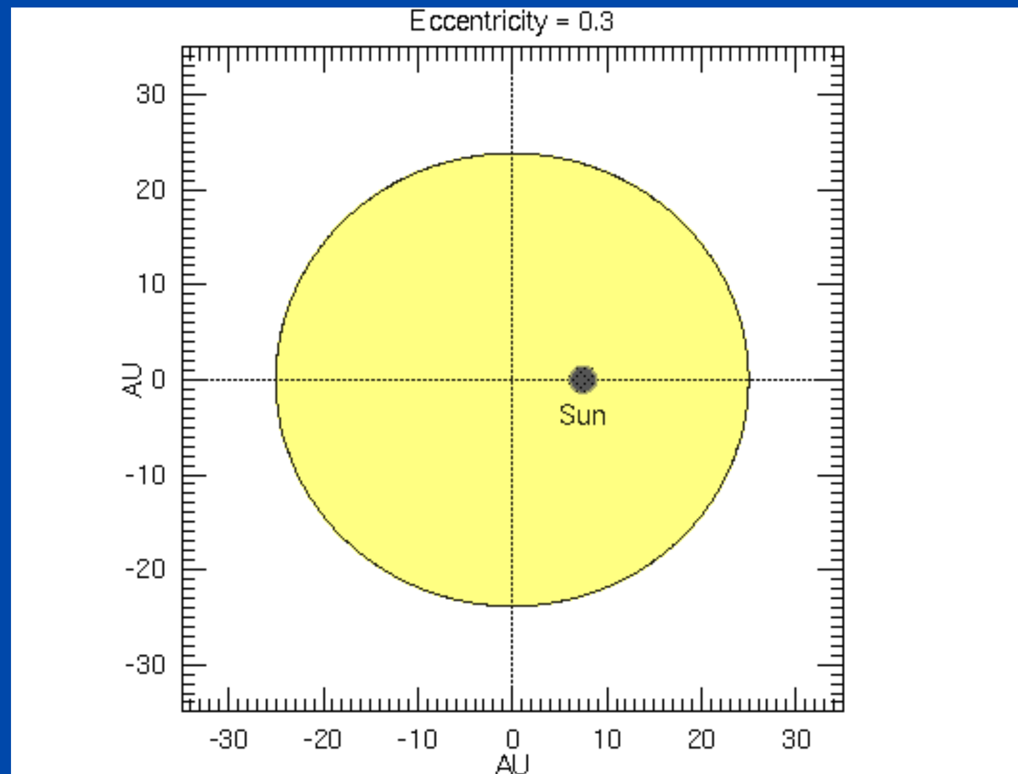
- The orbit of each planet around the Sun is an ellipse with the Sun at one focus



Drawing an **ellipse**: loop string around thumb tacks at each **focus** and stretch string tight with a pencil while moving the pencil around the tacks. The Sun is at one focus.

Difference between ellipse and circle

- **Eccentricity:**
  - (distance from center to focus) ÷ (semi-major axis)
- **Eccentricity = 0:**
  - a circle
- **Eccentricity = 1:**
  - a very flat oval
- **Ellipse is specified by its semi-major axis and eccentricity**



Ellipses with the **same** *semi-major axis* (25 AU) but **different** *eccentricities*. As the eccentricity increases, the Sun (at a *focus*) moves farther from the center. The difference between the *perihelion* and *aphelion* increases as well. The ellipse with eccentricity=0.3 looks almost circular. All of the planet orbits have eccentricities **less than** 0.3—this is why it was so hard to discover that planet orbits are ellipses, rather than circles.

More about ellipses

- The equation of an ellipse, centered on the point (0,0) which passes through the points (a,0) and (0,b) and whose major and minor axes are parallel to the x and y axes is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

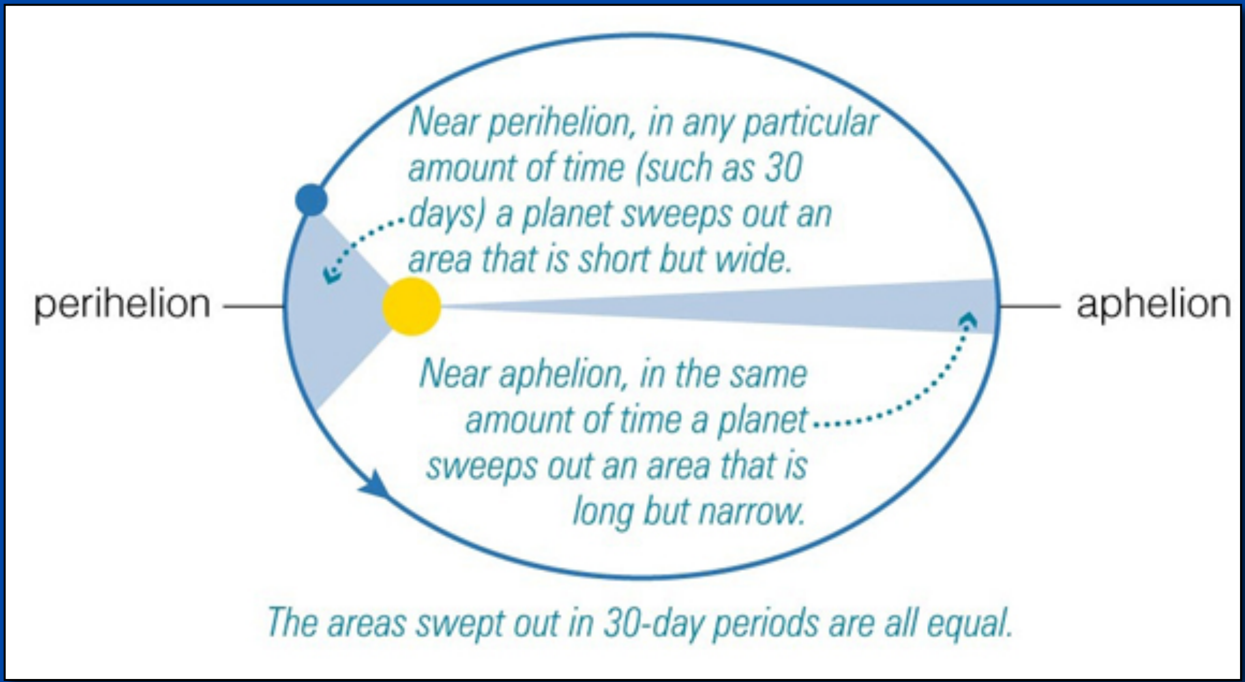
- The eccentricity is

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

- Ellipse applet

Kepler's second law

- As a planet moves around its orbit, it sweeps out equal areas in equal times
- Consequence: planet or comet moves fastest when it is closest to Sun
  - Kepler did not have a way to explain WHY



Speed of a planet in an elliptical orbit

- Moves fastest when close to Sun, slowest when farther away
- Intuitively: gravity is weaker when farther from the Sun
- Velocity applet

Kepler's third law

- Describes how a planet's orbital period (in years) is related to its average distance from the Sun (in Astronomical Units, or AU)
  - Earth-Sun average distance is 1 AU

(orbital period in years)^2 = (average dist. in AU)^3

$$p^2 = a^3$$

The more distant a planet is from the Sun, the longer its orbital period.

- Distant planets move at slower speeds, because gravity is weaker the farther you are from the Sun

Many good web sites for interactive animations of Kepler's laws

- http://astro.unl.edu/naap/pos/animations/kepler.swf
- http://highered.mcgraw-hill.com/olcweb/cgi/pluginpop.cgi?it=swf::800::600::/sites/dl/free/0072482621/78778/Kepler_Nav.swf::Keplers+Second+Law+Interactive
- Try out at least one of these (or find a better one on the web).
- As part of your next homework assignment, write a paragraph or two describing how you played with the animation, what you learned, and what surprised you the most.

ConcepTest 1

- Kepler's 3rd Law (that the period squared is proportional to the semi-major axis cubed) does NOT apply to the motion of

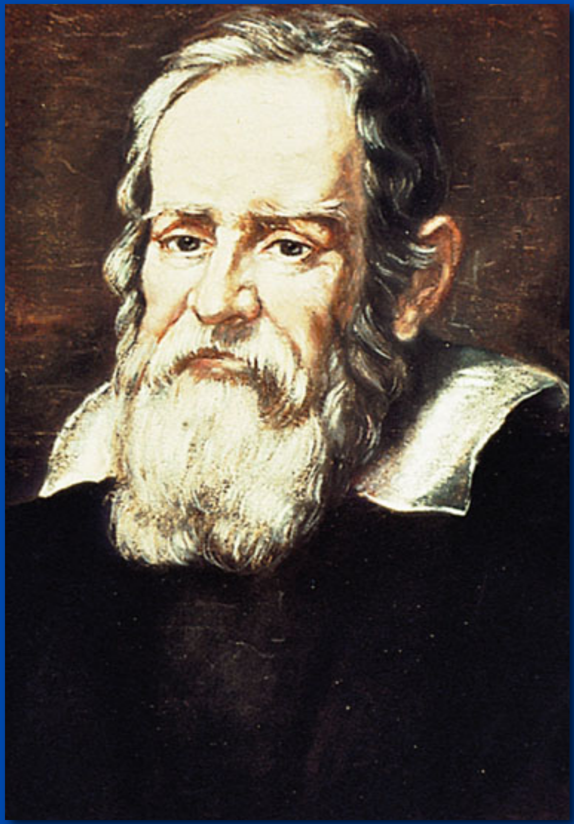
a) a satellite around the Earth

b) one star around another in a binary system

c) one galaxy around another

d) all of the above apply

How did Galileo make a stronger case for the Copernican revolution?



Galileo (1564-1642) overcame 3 major objections to Copernican view:

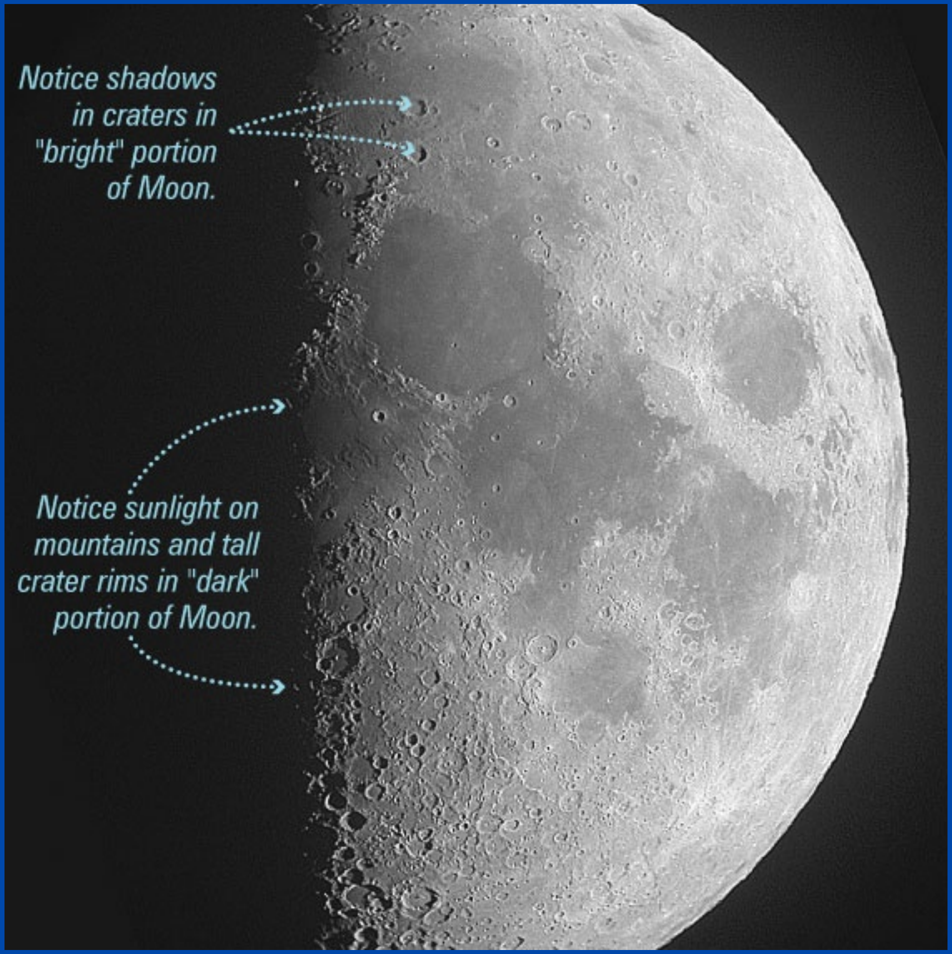
1. Earth could not be moving because objects in air would be left behind.
2. Non-circular orbits are not "perfect" as heavens should be.
3. If Earth were really orbiting Sun, we'd detect stellar parallax.
   - We would see nearby stars appear to move relative to background stars.

Overcoming the first objection (nature of motion)

Galileo's experiments showed that objects in air would stay with a moving Earth.

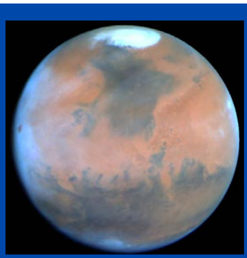
- Aristotle thought all objects naturally come to rest.
  - Experience based on horses pulling a heavy cart over a rutted ancient road!
- Galileo showed that objects will stay in motion unless a force acts to slow them down
  - Became Newton's first law of motion

Overcoming the second objection (heavenly perfection)



- Using his telescope, Galileo saw:
  - Sunspots on Sun ("imperfections")
  - Mountains and valleys on the Moon (proving it is not a perfect sphere)

Overcoming the third objection (parallax)



- Tycho thought he had measured stellar distances, so lack of parallax seemed to rule out an orbiting Earth.

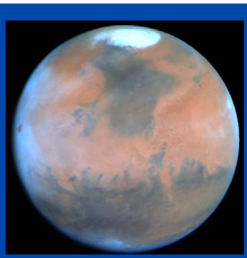
- Galileo showed that stars must be much farther away than Tycho thought — by using his telescope to see that the Milky Way is countless faint but individual stars.

✓ If stars were much farther away, then lack of detectable parallax was no longer so troubling.

Galileo's observations of the moons of Jupiter were key

- Galileo saw four moons orbiting Jupiter, proving that not all objects orbit the Earth.
- Felt that this justified the hypothesis that the Earth orbits the Sun.
- He kept good notes and published his results!

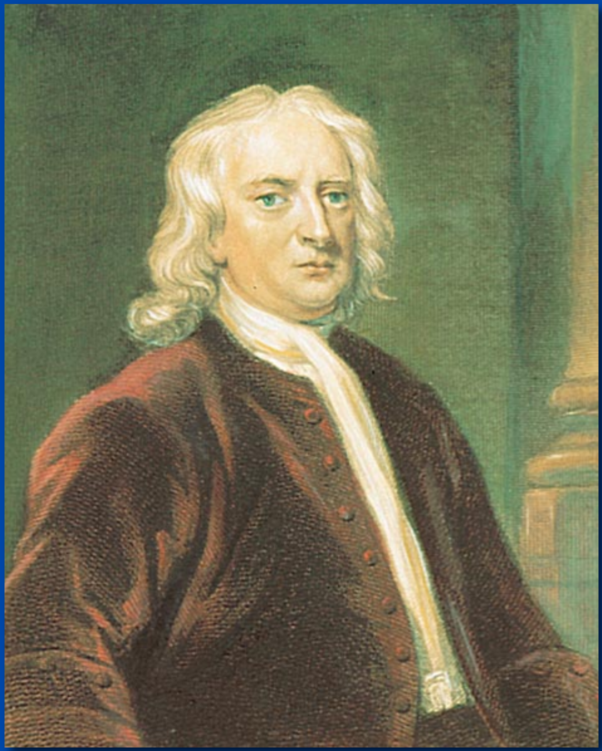
Galileo spent the last 8 years of his life under house arrest here:



Hills of Arcetri, near Florence

His daughter lived in a convent nearby, and his students visited him for scientific discussions

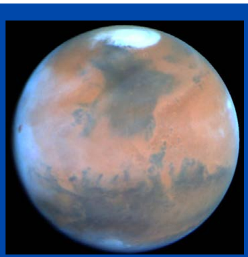
How did Newton change our view of the universe?

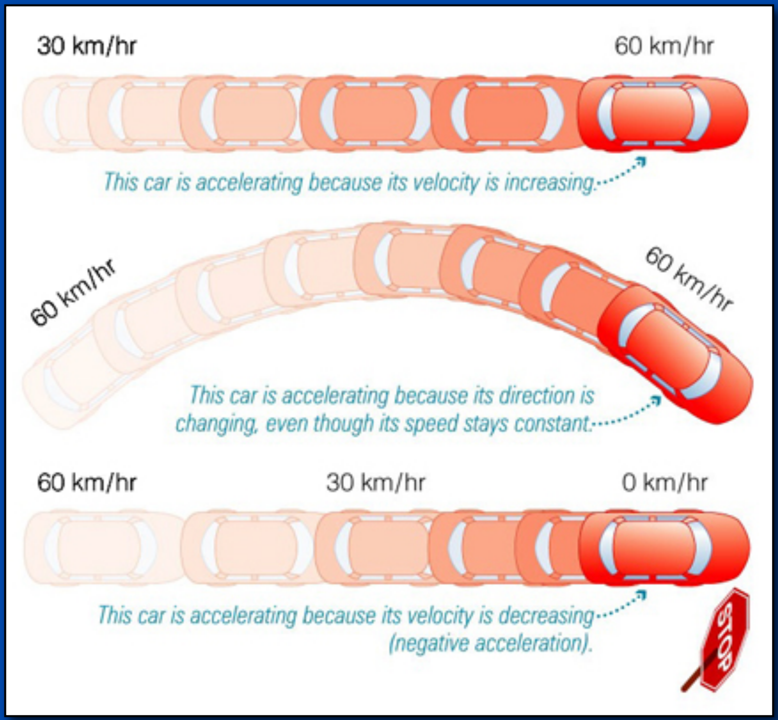


Sir Isaac Newton (1642-1727)

- **Realized that the same physical laws that operate on Earth also operate in the heavens**
  - ***one universe***
- **Discovered laws of motion and gravity**
  - **Explained *why* Kepler's laws work**
- **Much more: Experiments with light; first reflecting telescope, calculus…**

Three ways to describe motion





- **Speed: Rate at which object moves**

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

example: 10 meters/sec

- **Velocity: Speed and direction**
  example: 10 meters/sec, due east

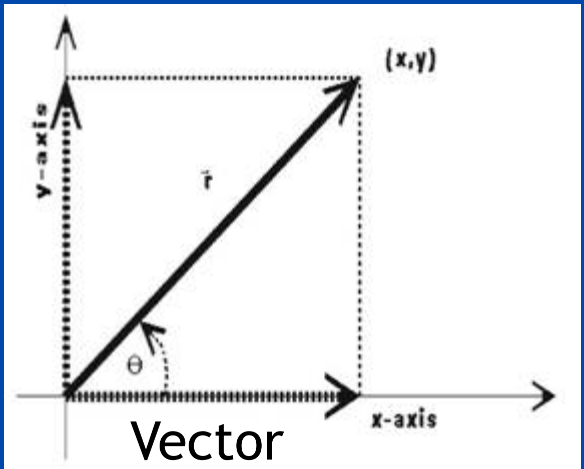
- **Acceleration: Change in velocity**
  example: speed/time (meters/sec^2) with direction

First, some definitions and concepts

- Speed: rate of change of distance with time

  $$v = \Delta|x| / \Delta t = dx / dt$$

  - km/sec, miles/hour
  - furlongs per fortnight

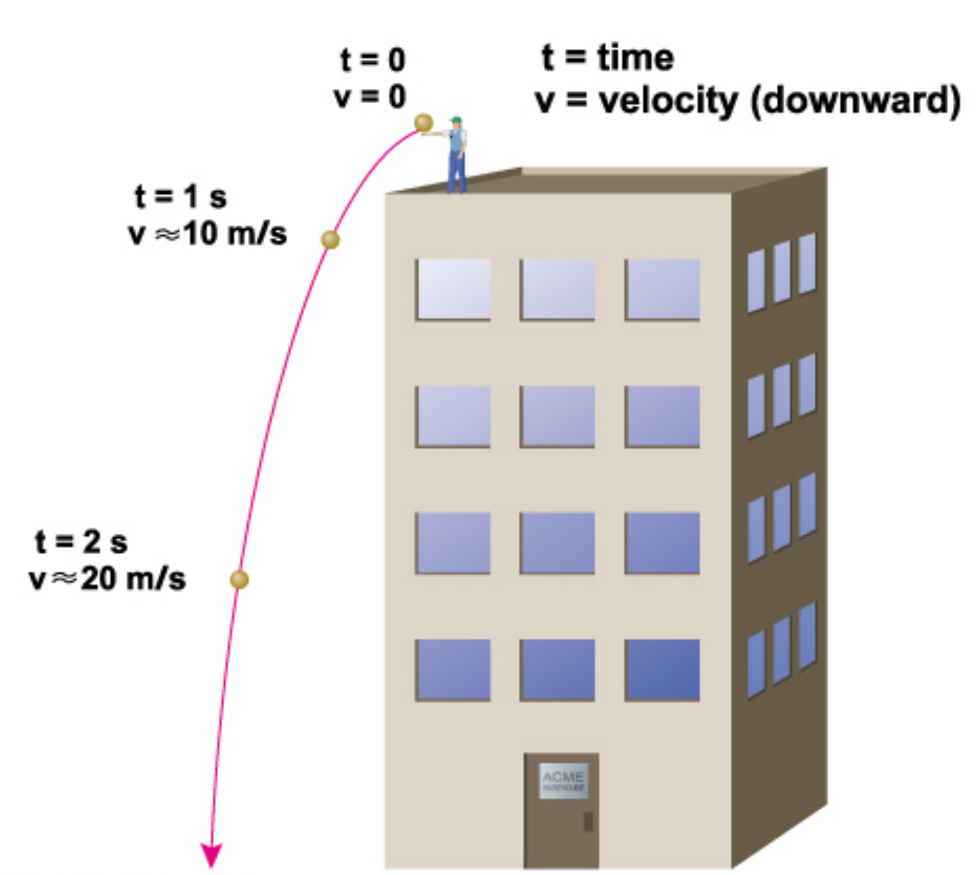


- Velocity: speed in a given *direction*
  - $\vec{v} = \Delta\vec{x} / \Delta t = d\vec{x} / dt$
  - where $\vec{x}$ and $\vec{v}$ are vectors
  - a vector quantity has a magnitude and a direction

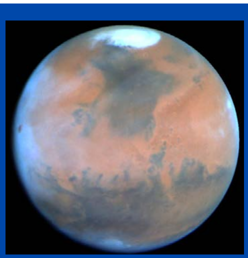
- Acceleration: rate of change of velocity with time
  - $\vec{a} = \Delta\vec{v} / \Delta t = d\vec{v} / dt = d^2\vec{x} / dt^2$
  - units: km/sec^2, m/sec^2, $miles/hour^2$ in a specific direction
  - in other words, change in (miles/hour) per hour

The Acceleration of Gravity

- All falling objects accelerate at the same rate (not counting friction of air resistance).
- On Earth, acceleration of gravity is $g \approx 10$ meters/sec^2
  - Speed increases by 10 meters/sec with each second of falling.

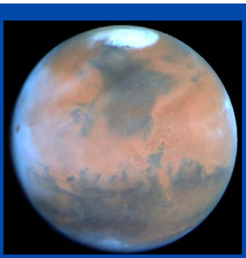


The Acceleration of Gravity (g)



- Galileo showed that *g* is the *same* for all falling objects, regardless of their mass.
  - Dropped objects from the leaning tower of Pisa
- Heavy bodies fall the same way as light bodies (if friction doesn't matter)

The Acceleration of Gravity (g)



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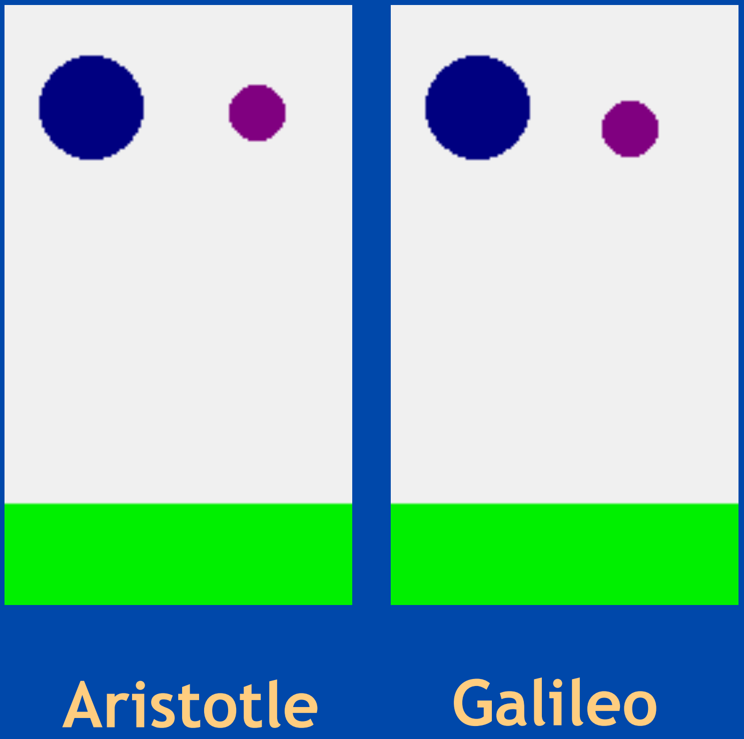
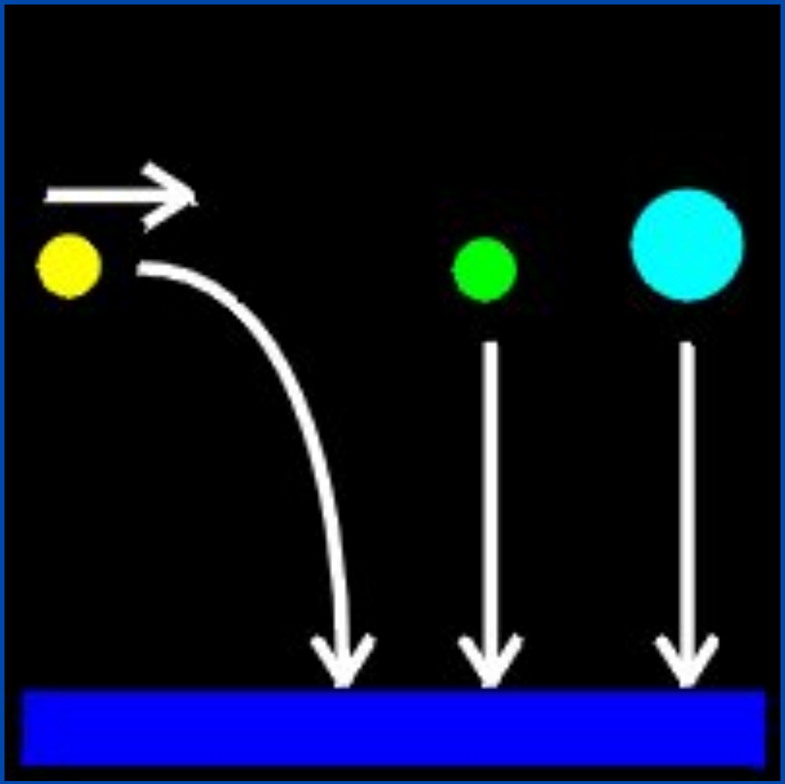
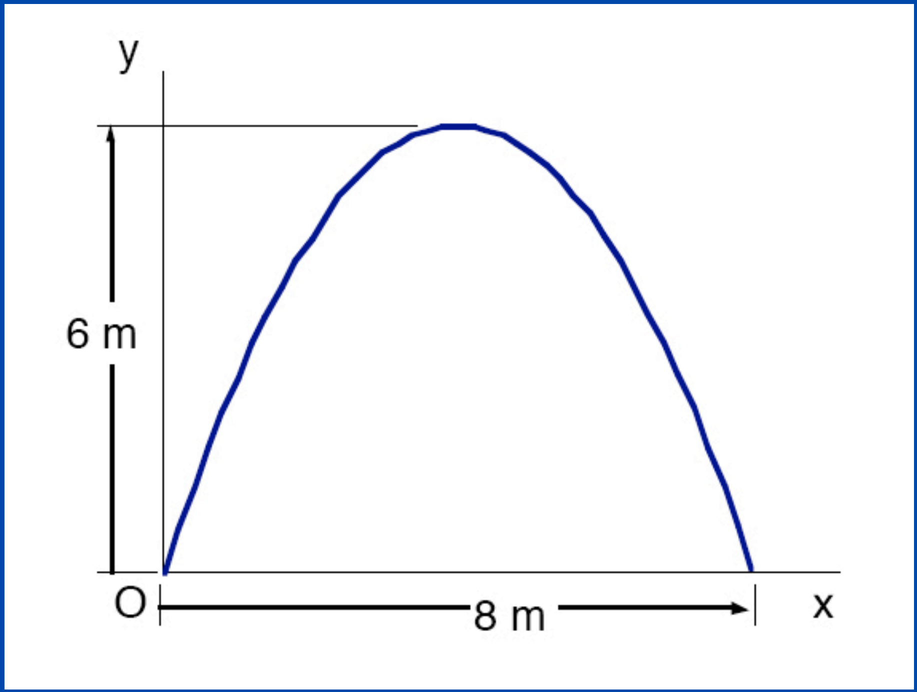


Galileo said all three of these balls will hit the ground at the same time

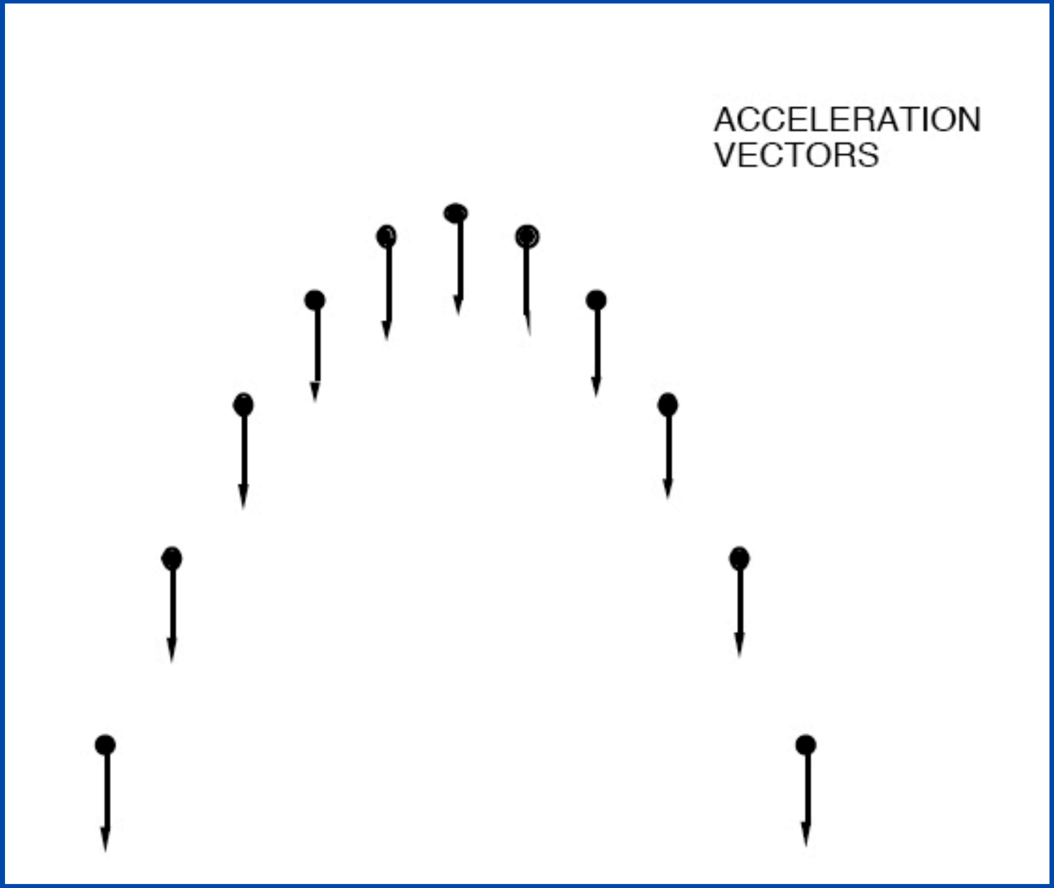


Trajectory when you throw a ball up into the air

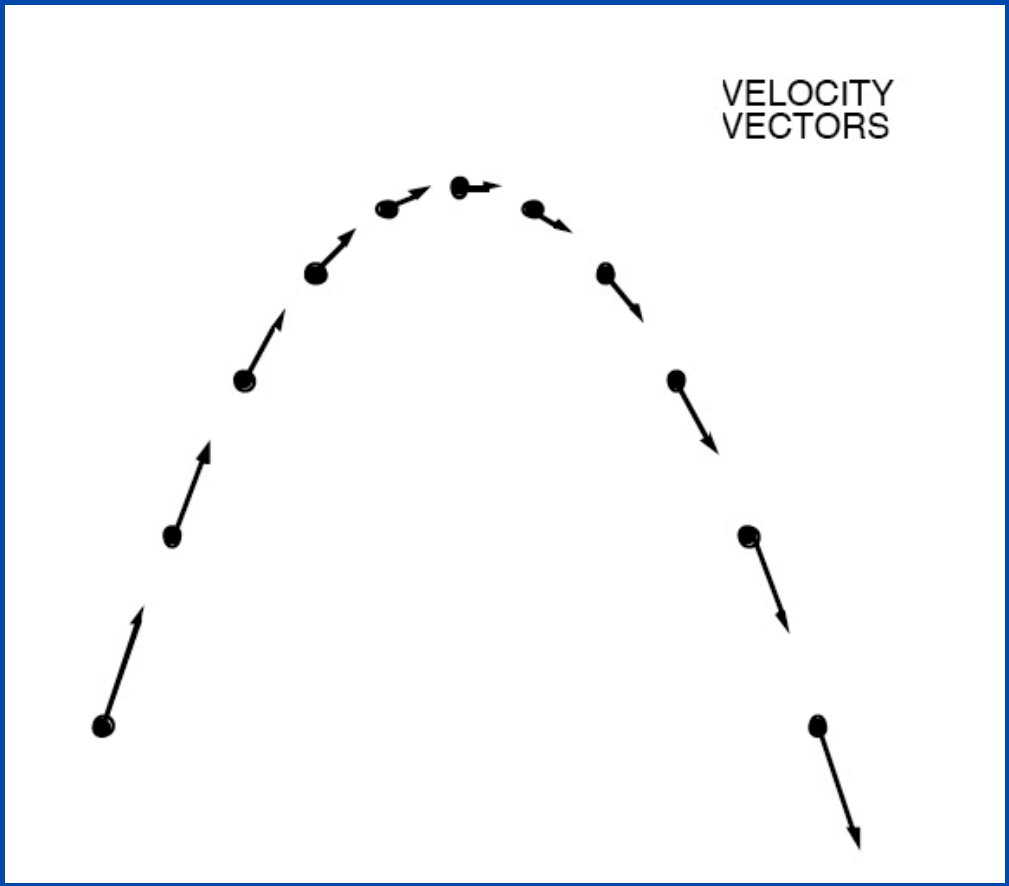


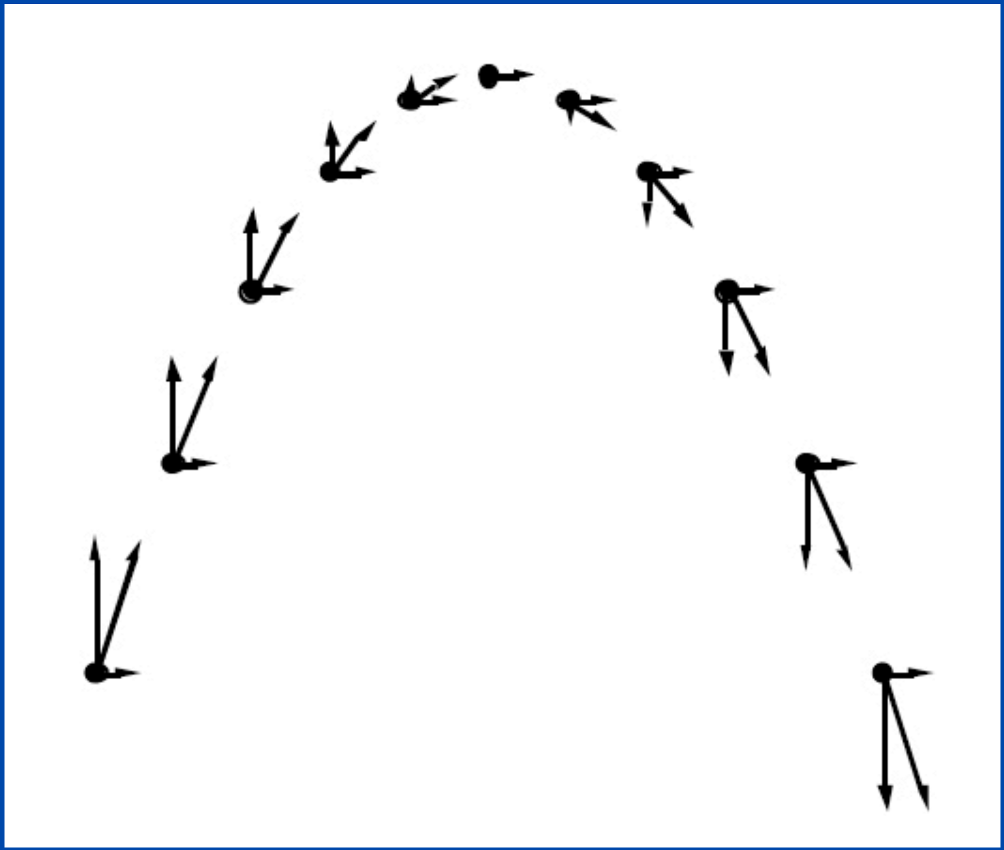
- Path is in shape of a parabola
- First goes up, but more and more slowly
- Then turns around and falls down to the ground
- Horizontal speed stays constant

Acceleration due to gravity always points downwards towards ground



Velocity changes throughout the fall

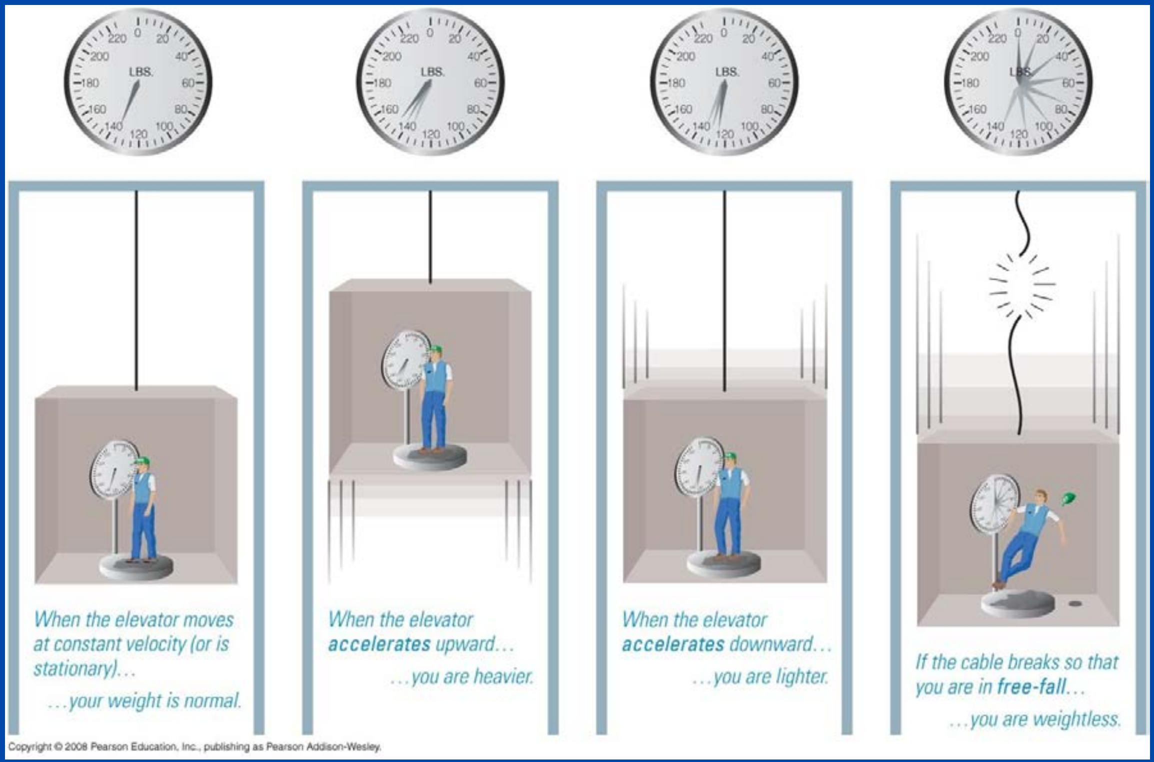




- Horizontal velocity stays constant
- Vertical velocity continually decreases due to gravitational acceleration downwards

How is mass different from weight?

- Mass - the <u>amount of matter</u> in an object
- Weight - the <u>force</u> that acts upon an object



- You are weightless in free-fall!
- You fall at same rate as the elevator

Concept Test 2

On the Moon:

A. My weight is the same, my mass is less.

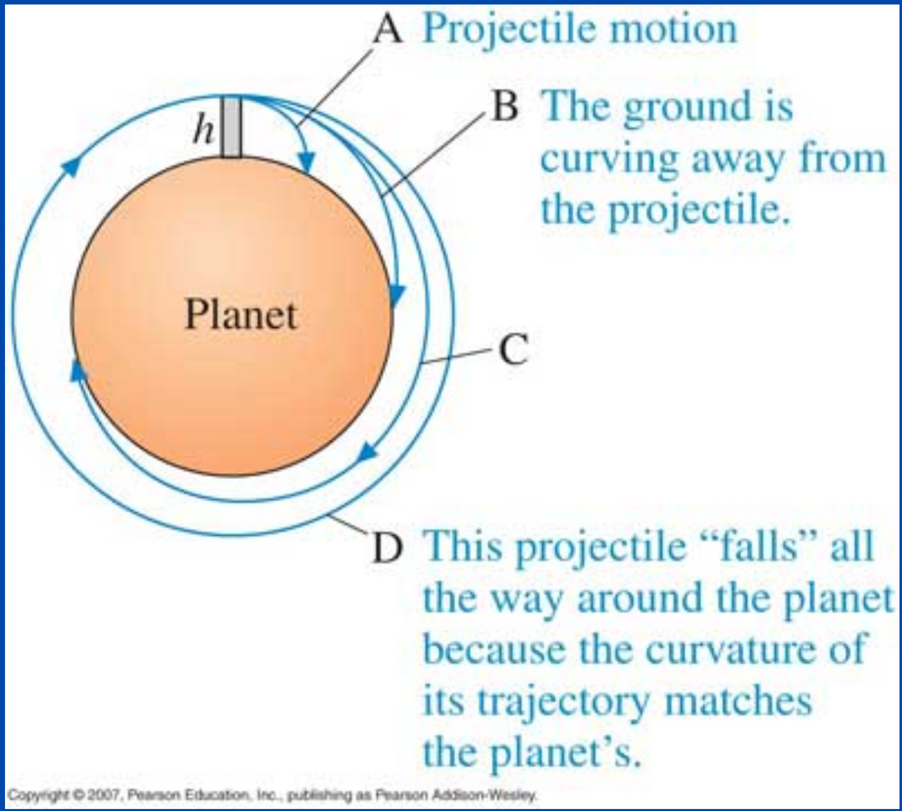
B. My weight is less, my mass is the same.

C. My weight is more, my mass is the same.

D. My weight is more, my mass is less.

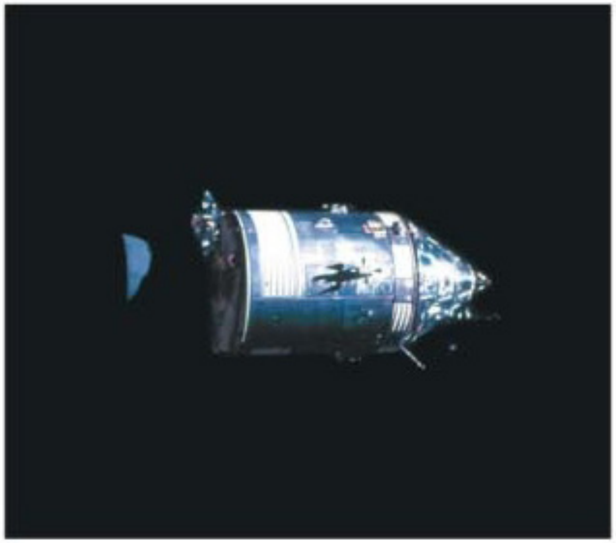
Why are astronauts weightless in space?

- There *is* gravity in space
- Weightlessness is due to a constant state of free-fall



http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/NewtMtn/NewtMtn.html

What are Newton's three laws of motion?



Example: *A spaceship needs no fuel to keep moving in space.*

Newton's first law of motion:

An object moves at constant velocity unless a net force acts to change its speed or direction.

Newton's second law of motion

Example: *A baseball accelerates as the pitcher applies a force by moving his arm. (Once the ball is released, the force from the pitcher's arm ceases, and the ball's path changes only because of the forces of gravity and air resistance.)*

Force = mass X acceleration

$$\vec{F} = m\vec{a}$$

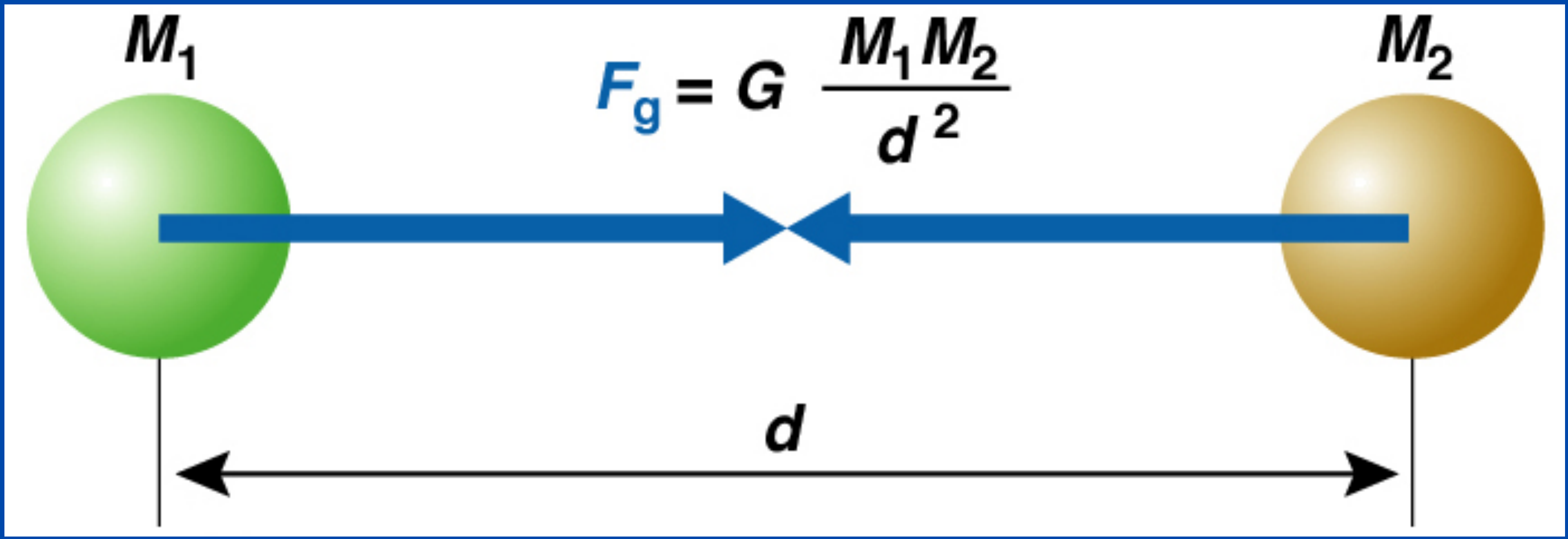
Newton's third law of motion:

Example: *A rocket is propelled upward by a force equal and opposite to the force with which gas is expelled out its back.*

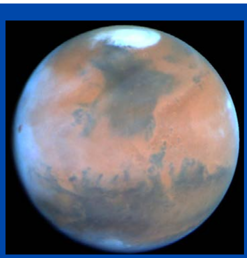
For every force, there is an *equal and opposite* reaction force.

Newton's Universal Law of Gravitation

1. Every mass attracts every other mass.
2. Attraction is *directly* proportional to the product of their masses.
3. Attraction is *inversely* proportional to the *square* of the distance between their centers.



Newton's universal law of gravitation

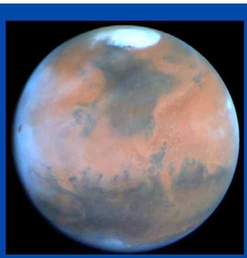


- Force of gravity between two bodies, 1 and 2

$$F_{gravity} = G\left(\frac{m_1 m_2}{d^2}\right)$$

- G is a constant of nature, the "gravitational constant"
- m_1 and m_2 are the masses of two bodies whose gravity we are considering
- d is the distance between the two bodies

Implications of "inverse square law"

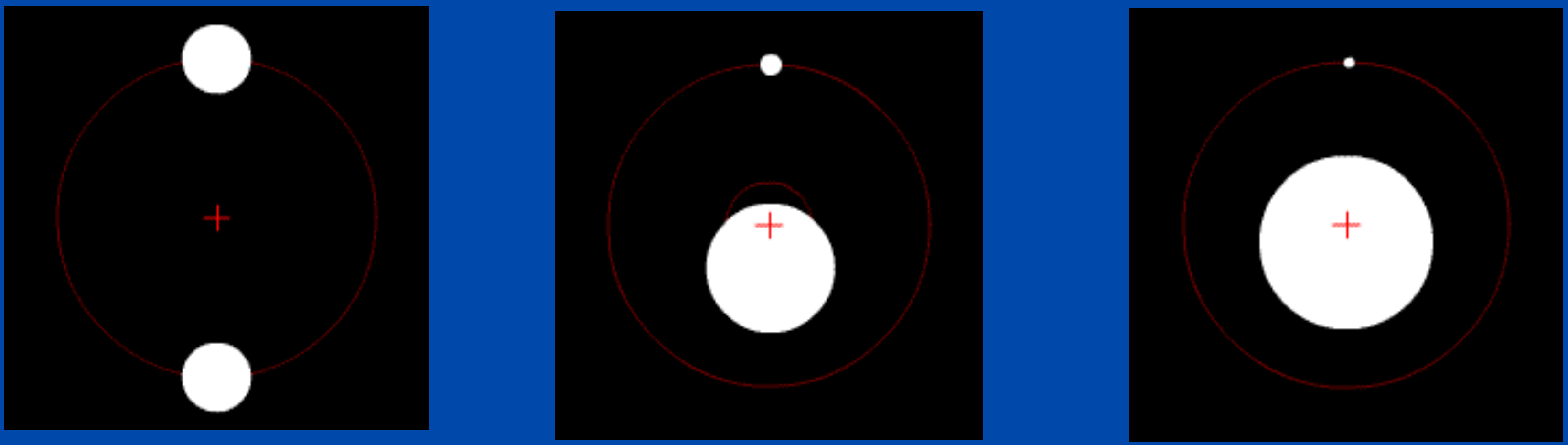


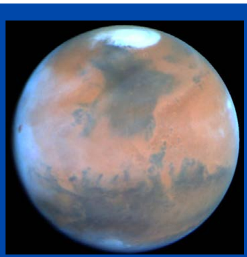
$$F_{gravity} = G\left(\frac{m_1 m_2}{d^2}\right) \propto d^{-2}$$

- Gravitational force decreases as you get farther away from an object
- Falls off as the <u>square</u> of the distance away
- If you go twice as far away, force is 4 x smaller

Center of Mass

- Because of momentum conservation, two objects orbit around their “center of mass”





Why do all objects fall at the same rate?

$$a_{\text{rock}} = \frac{F_{\text{g}}}{M_{\text{rock}}} \qquad\qquad F_{\text{g}} = G\,\frac{M_{\text{Earth}} M_{\text{rock}}}{R_{\text{Earth}}^2}$$

$$a_{\text{rock}} = \frac{F_g}{M_{rock}} = \left(G\,\frac{M_{\text{Earth}} \cancel{M_{\text{rock}}}}{R_{\text{Earth}}^2} \right) \Big/ \cancel{M_{\text{rock}}} = G\,\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}$$

- **The gravitational acceleration of an object like a rock does not depend on its mass because M_{rock} in the equation for acceleration cancels M_{rock} in the equation for gravitational force**

Acceleration due to gravity at the surface of the Earth

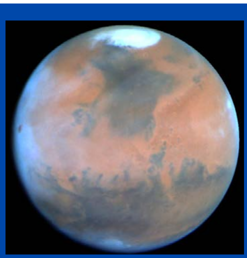
- m_1 = mass of Earth, d = radius of Earth R_E

$$F_{gravity} = G\left(\frac{m_1 m_2}{d^2}\right) = \left(\frac{Gm_{earth}}{R_E^2}\right)m_2 = m_2 g$$

- At surface of Earth, acceleration due to gravity is

$$g = \left(\frac{Gm_{earth}}{R_E^2}\right) = 9.8 \text{ meters / sec}^2$$

Newton's version of Kepler's third law



$$p^2 = \left[\frac{4\pi^2}{G\left(m_1 + m_2\right)} \right] \times \; a^3$$

- In this form, applies to any pair of orbiting objects with period *p* and average separation *a*
- So Newton provided a physical reason why the period and semi-major axis of a planet are related

Remarkable consequence

- We can calculate mass of the Sun just by knowing the length of a year and the size of the Earth's orbit (150 million km)

$$(m_{Earth} + m_{sun}) \approx m_{sun} \text{ because } m_{Earth} << m_{sun}$$

$$\left(p_{Earth}\right)^2 = \frac{4\pi^2}{Gm_{sun}}\left(a_{Earth}\right)^3$$

$$m_{sun} = \frac{4\pi^2}{G}\frac{\left(a_{Earth}\right)^3}{\left(p_{Earth}\right)^2} = 2\times10^{30}\,kg = 2\times10^{33}\,g$$

ConcepTest 3

- You hold a ball in your hand at a fixed height and release it. Its initial acceleration after you let go is

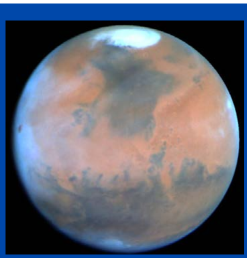
    1. Up
    2. Zero
    3. Down

- Why?

Newton's First Law of Gravity: What have we learned?

- What determines the strength of gravity?
  - Directly proportional to the *product* of the masses (m_1 x m_2)
  - *Inversely* proportional to the *square* of the separation
- How does Newton's law of gravity allow us to extend Kepler's laws?
  - Applies to other objects, not just planets.
  - Includes unbound orbit shapes: parabola, hyperbola
  - Can be used to measure mass of orbiting systems.

Newton's second law



- Definition: momentum = mass x velocity
  - $\vec{p} = m\vec{v}$
  - momentum has a <u>direction</u> because $\vec{v}$ does

- Newton's second law:

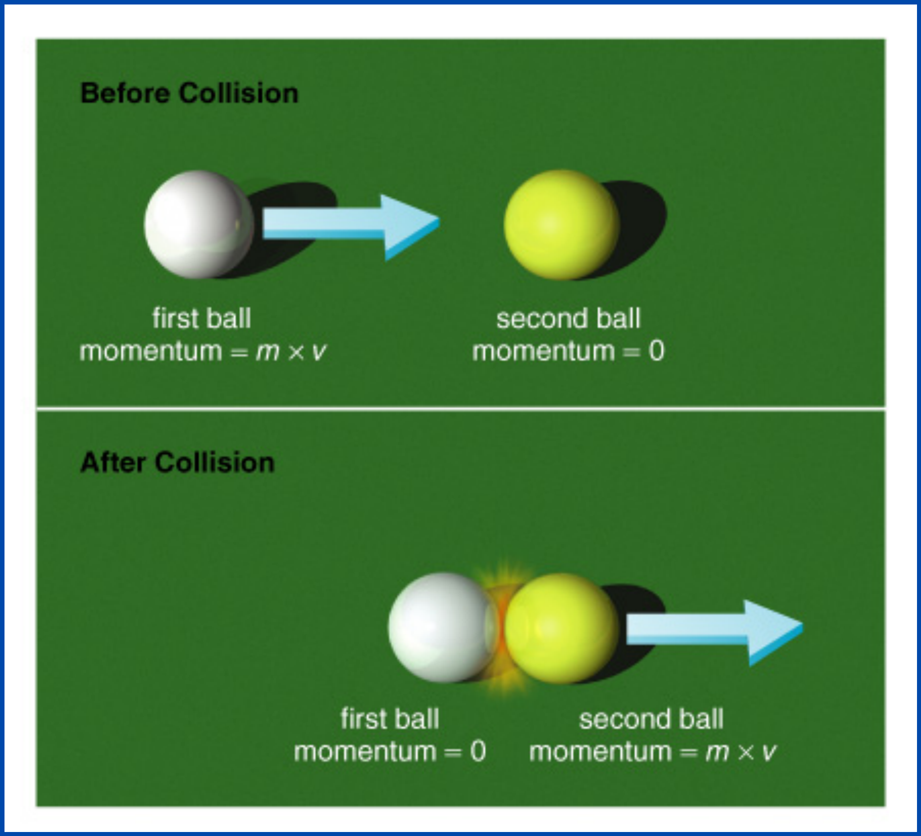
Force = mass x acceleration = rate of change of momentum

$$\vec{F} = m\vec{a} = \text{rate of change of } \left(m\vec{v}\right) \text{ with time} = \frac{\Delta(m\vec{v})}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t}$$

Consequences of Newton's second law

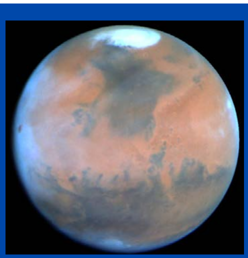
- F = m a
- If there is no force (F = 0), there is no acceleration (mv = momentum = constant)
- Momentum is conserved if there are no forces acting
  - With no force, objects move with constant velocity (!)
- If force is due to gravity, for example near surface of the Earth,
  - acceleration = acceleration due to gravity
  - a = g
  - Force on an object of mass m = m g

Conservation of Momentum



- The total momentum of interacting objects cannot change unless an external force is acting on them

Conservation of Angular Momentum

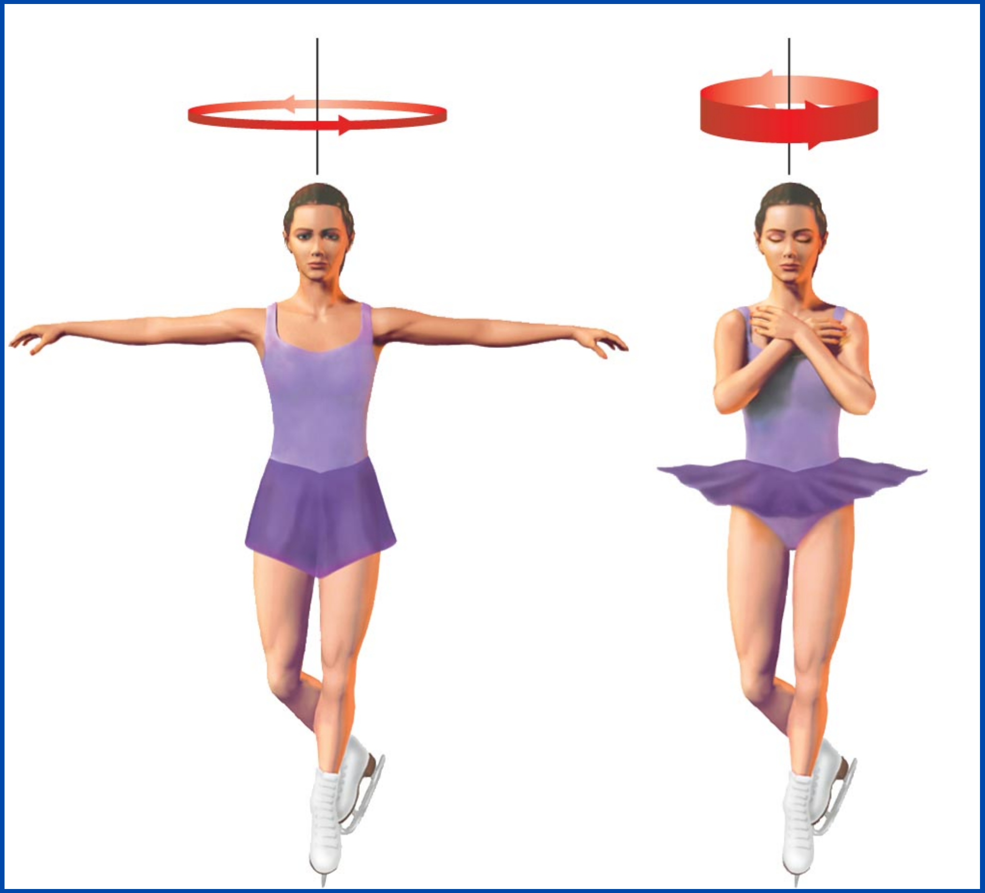


angular momentum = mass x velocity x radius

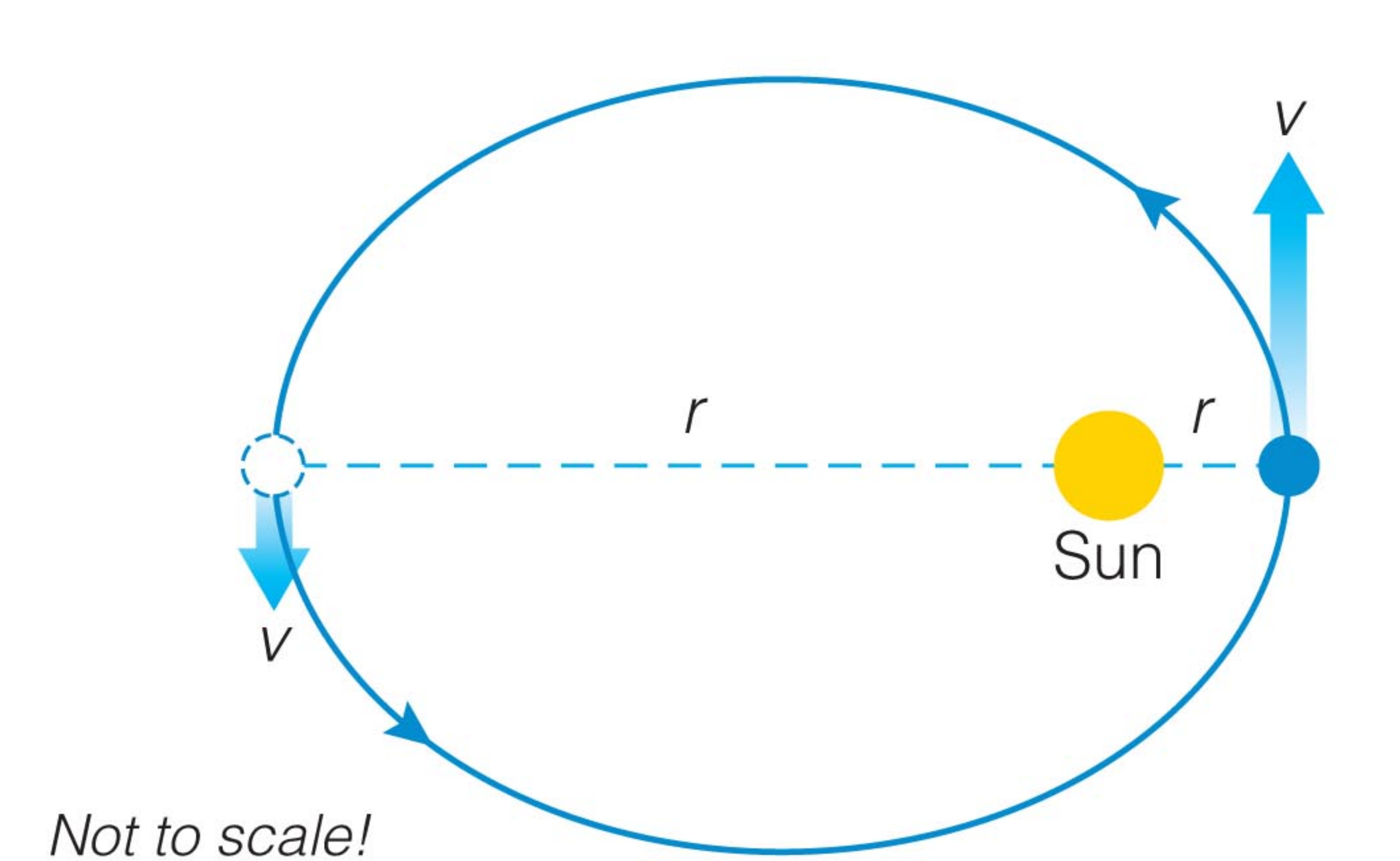
- **The angular momentum of an object cannot change unless an external twisting force (torque) is acting on it**
- **Earth experiences no twisting force as it orbits the Sun, so its rotation and orbit will continue indefinitely**
- **Intuitive explanation of Kepler's law: planets that are close to the Sun (small radius) must move faster in their orbits**

Angular momentum conservation also explains why skater spins faster as she pulls in her arms

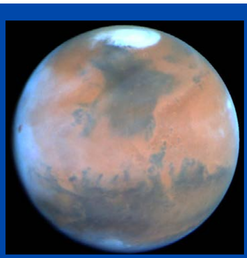
angular momentum = mass x velocity x radius



What keeps a planet rotating and orbiting the Sun?



See you Monday night at lunar eclipse and stargazing



Please check
http://www.astro.ucsc.edu/astronomy_club
before you come down to Music Center at about 10pm

Bring warm clothes and a flashlight